Chapter 3
Saving , Financial Market , and Economic Growth

3.1 The Cass – Koopmans – Ramsey model

3.1.1 Assumptions

- The follows the Solow model in assuming rational maximizing behavior.
- It drops the assumption of a fixed saving rate and adds a utility function to the model.
- People as rational consumers who determine their saving behavior by weighing the alternatives of both consuming today and saving today and reaping the benefits of higher output and income in the future. Hence, saving can be determined endogenously.
- Competitive firms rent capital and hire labor to produce and sell output.
- A fixed number of infinitely- lived household supply labor, hold capital, consume and save.
- All economic agents are homogenous.

A) Production

- There are a large number of identical firms. Each has access to the production function **Y=F(K,L)**.
- The production function exhibits constant returns to scale and diminishing returns to individual factors.
- Diminishing returns to labor cause the return to capital to increase when the population grows, and this increase in the return to capital provides the incentives to save and reap the benefits from investment.
- The firm hire workers and rent capital in competitive factor market and sells their output in a competitive output market.

- Any profit the firms earned are owned by the households.

- Assume that the economy produces just one good either consumed or invested:

$$Y_t = F(K_t, N_t) = C_t + \frac{dK_t}{dt}$$

where the population N_t grows at rate n

=> $$Y_t = C_t + N_t \frac{dK}{dt} + k\frac{dN}{dt} \quad \text{since} \quad \frac{K}{L} = k \quad \text{and} \quad K = kL .$$

- Put into per capita terms:

$$\frac{Y}{N} = \frac{C}{N} + \frac{dk}{dt} + \frac{k}{N} \cdot \frac{dN}{dt} \quad => \quad Y_t = c + \frac{dk}{dt} + nk$$

=> $$f(kt) = c_t + \frac{dk}{dt} + nk_t \quad => \quad \dot{k} = f(k) - nk - c \quad \text{.....................} (1)$$

B) Household behavior

- People live in households and they seek to maximize the long - run welfare. The model includes people's desire to leave bequests in the welfare - maximization problem.

- Present members of the household discount future welfare. If given the choice, ceteris paribus, they would prefer to enjoy a certain level of welfare today rather than have an equal level of welfare tomorrow.

- Those presently alive to tradeoff between increased personal welfare and diminished welfare for future hears.

- Each household is assumed to grow in size over time, and thus rational individuals who care about their offspring will have to be concerned about equipping additional household members with enough capital to maintain per capita income over time.

- The model assumes that household live forever, Robert Barro says that if people take their offspring's welfare as their own, it is appropriate to model them as if they live forever.

- There are many identical households in the economy. Each household contains **Li** members.

- Household are assumed to earn income by providing labour services in exchange for wages, **W**, and earning interest income, **r**, on assets, denominated as **B** (for bonds).

- Household use their income, **wL+rB** to buy consumption goods, **C**, and save by purchasing more assets (bonds), denoted as the change in **B**, or Δ**B**.

- Total saving by the **i**th household in each period **t**

$$Si_t = \Delta Bi_t = W_t Li_t + r_t Bi_t - Ci_t \text{.......................} (2)$$

- Put into per capita terms:

$$\Delta bi_t = w_t + r_t bi_t - ci_t - nb_t \text{.......................} (3)$$

The term nb_t is added in **(3)** because as the number of household members grows at the rate of **n** over time, existing holdings of assets must split among more people. Hence, the per capita holding of assets decreases at the rate **n**.

C) Solving the maximization problem

- Maximization problem: max $u(0) = \int_0^\infty u(ct)\, e^{-\theta t}\, dt$ .... **(4)**

Subject to $\dot{k} = f(k) - nk - c$ .... **(1)** and $k_0 > 0$

Where: **u(.)** = the household's total instantaneous utility at time **t**

θ = the positive rate of time preference or the discount rate

u(0) = a measurement of utility at **t = 0**

- **(4)** describes current utility as a weighted average of all future flows of per capita utility **u(ct)**.

- With a positive of θ, the minus sign in front of θt implies that future consumption is valued less than present consumption.

- Specify the instantaneous utility function:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \text{ and } \gamma > 0, \quad \gamma \neq 1 \text{ ....} (5)$$

- **(5)** is a constant international elasticity of substitution utility function.

- γ determines the household's willingness to shift consumption between periods.

- $1/\gamma$ is the elasticity of substitution between consumption at any two pants in time.

- The higher is γ, the more rapidly marginal utility falls as consumption rises, and so the less people are willing to let consumption vary over time.

- The smaller is γ, the more slowly marginal utility fall consumption rises, and so consumption will vary a lot in response to changes in the differences between the rate of return on savings and the rate θ at which people discount the future.

- If γ is close to one, utility function can be written as $\ln c$

  Proof:

$$\frac{c^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma} = \frac{c^{1-\gamma}-1}{1-\gamma}$$

=> $\lim_{\gamma \to 1} \frac{c^{1-\gamma}-1}{1-\gamma}$ provided type $\frac{0}{0}$ using L' Hopital's rule

=> $\lim_{\gamma \to 1} \frac{c^{1-\gamma} \ln c(-1)}{-1} = \ln c$

- If γ is close to zero, this utility function is close to linear in c, and marginal utility is also nearly constant, an so people are vary willing to substitute one year's consumption for another year's consumption.

- The elasticity of substitution over time is assumed to be constant.

- The solution to the inter-temporal maximization problem.

$$\frac{\dot{c}}{c} = \sigma\left[f'(k) - n - \theta \right] \text{.......................................} \textbf{(6)}$$

$\uparrow$

instantaneous elasticity of substitution y. consumption at two points in time

It determines the rate of growth of consumption that maximizes utility over time.

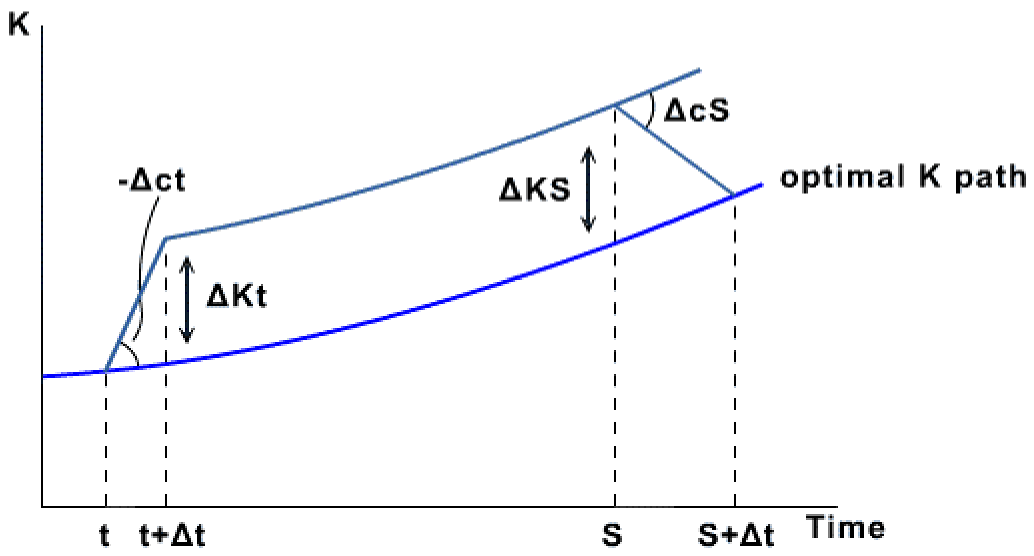
3.1.2 The Keynes – Ramsey rule

- Along the optimal path, small reallocations in consumption must leave welfare unchanged so that the loss in utility at time **t** must be equal to the discounted increase in utility at time **s**.

- Consider two point in time, **t** and **s**, where **s > t** . Decrease c_t by a small amount Δc_t at time **t** for a period of length Δt, thus increasing capital accumulation by $\Delta c_t \Delta t$.

- All the increased capital is consumed during an interval length Δt starting at **s**, with consumption being unchanged from the level on the original path.

- If the household is optimizing, the marginal in part of this change on lifetime must be zero.



3.1.3 The balanced growth path

A) Properties of the balanced growth path

- Capital, output and consumption per capita are constant. Since $\mathbf{y}$ and $\mathbf{c}$ are constant, the saving rate in the steady is also constant and depends on the constant rate of population growth and the steady – state level of $\mathbf{k}$.

$$\text{Saving rate}\quad \frac{s^*}{y^*} = \frac{y^* - c^*}{y^*}$$

- The CKR model lets welfare – maximizing households endogenously set their level of saving, but it also generates a constant long-run saving rate.

- The total capital stock, total output and total consumption grow at rate $\mathbf{n}$.

B) The balanced growth path and the modified golden – rule level of a capital

- From **(6)**, with $\mathbf{dc/\,dt = 0}$ and $\sigma = \mathbf{1}$, we have the modified golden rule relationship:

$$f'(k) - n - \theta = 0 \quad \Rightarrow \quad f'(k^*) = n + \theta \qquad \textbf{(7)}$$

- The modified golden rule condition implies that the capital stock is reduced below the golden rule level by an amount that depends on the rate of time preference.

- The saving rate is too low, the steady state $\mathbf{k^*_{MGR}}$ is less than the Golden-rule level of $\mathbf{k^*_{GR}}$, and the long-run level of consumption $\mathbf{c^*_{MGR}}$ is less than $\mathbf{c^*_{GR}}$. Because consumers discount future levels of consumption, they save too little.

- Because household value present consumption more than future consumption, the benefit of the eventual permanent increase in future consumption is limited:

- With $\sigma = \mathbf{1}$, **(6)** becomes: $\dfrac{\dot{c}}{c} = r - n - \theta$ **(8)**

- The higher the $\mathbf{MP_k}$ relative to the rate of preference, with constant the elasticity of substitution, the more it pays to depress the current level of consumption in order to enjoy higher consumption later. Consumption will be increasing over time on the optimal path.

$$\frac{\dot{c}}{c} > 0 \quad => \quad r - n > \theta \quad => \quad \text{increasing growth rate}$$

$$\frac{\dot{c}}{c} = 0 \quad => \quad r - n = \theta \quad => \quad \text{constant growth rate}$$

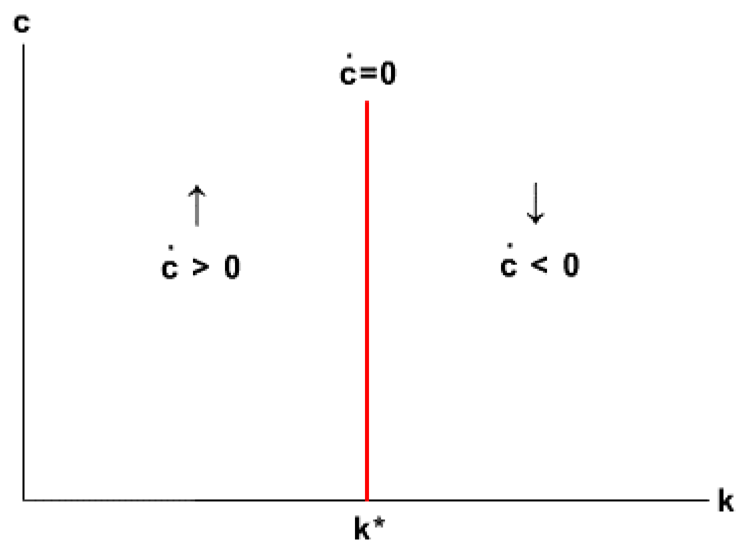
$$\frac{\dot{c}}{c} < 0 \quad => \quad r - n < \theta \quad => \quad \text{decreasing growth rate}$$

3.1.4 The dynamics of the economy

A) The dynamic of c

- Consumption is increasing to the left of the $\dot{c} = 0$ locus, where $f'(k) > n + \theta$, and decreasing to the night of the $\dot{c} = 0$ locus.

- $\frac{d\dot{c}}{dk} = f''(k) < 0 \quad => \quad dk \uparrow \rightarrow dc \downarrow$



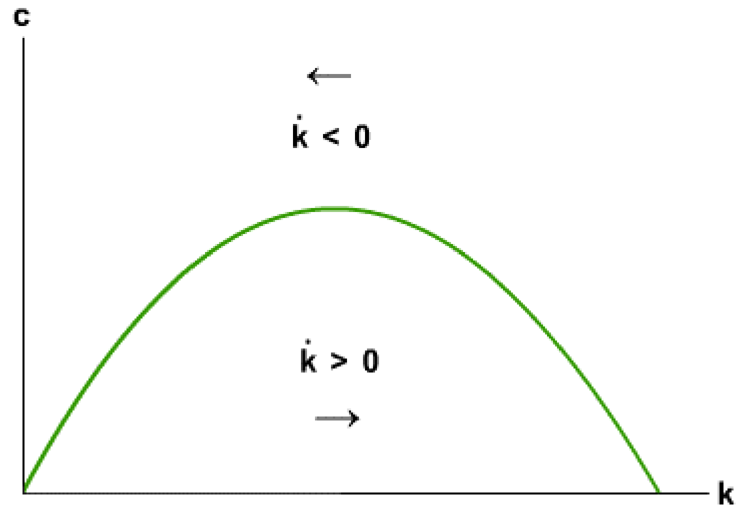
- c is rising if $k < k^*$ and falling if $k > k^*$. The $\dot{c} = 0$ line at $k = k^*$ shows c is constant for this value of k.

B) The dynamic of **k**

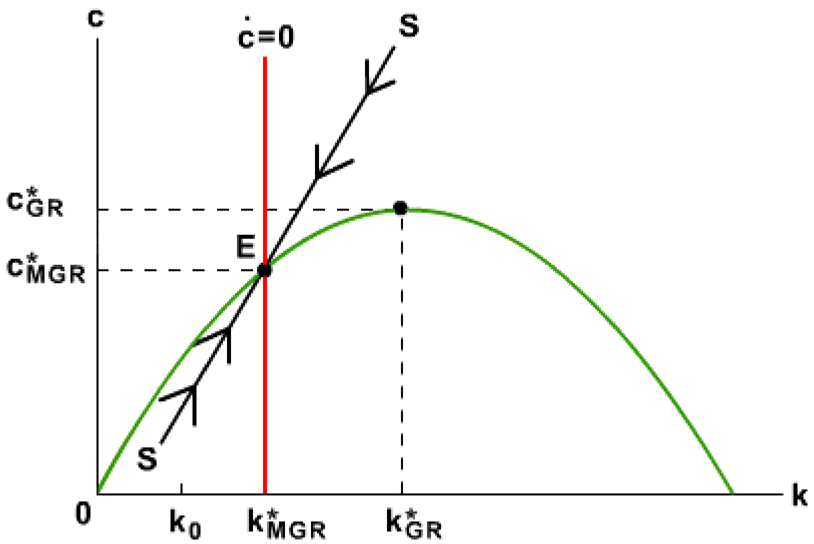
- When consumption **c** exceeds the level that would just maintain **k** constant ($\dot{k} = 0$), the capital – labor ratio **k** is falling.

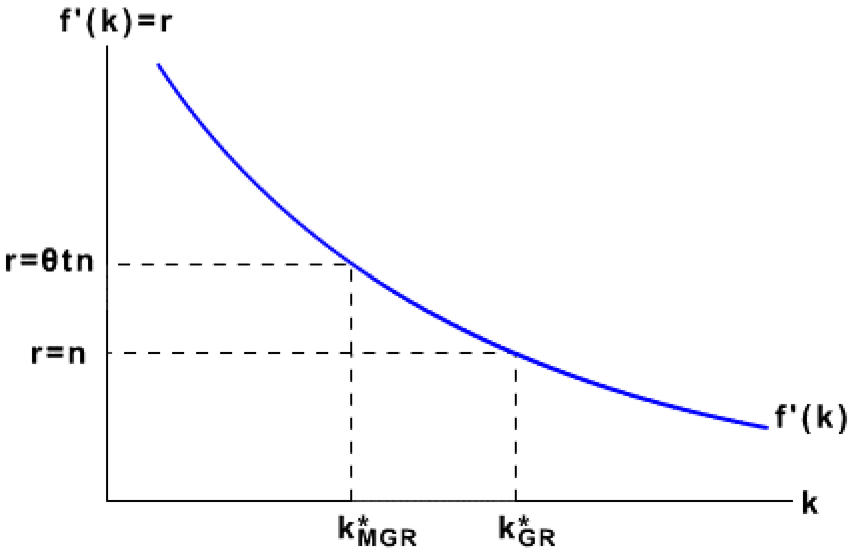
- When consumption **c** is less than the level that yields $\dot{k} = 0$, capital – labor ratio **k** is rising.

$$\frac{d\dot{k}}{dc} = -1 < 0 \quad \Rightarrow dc \uparrow \rightarrow dk \downarrow$$



C) The phase diagram

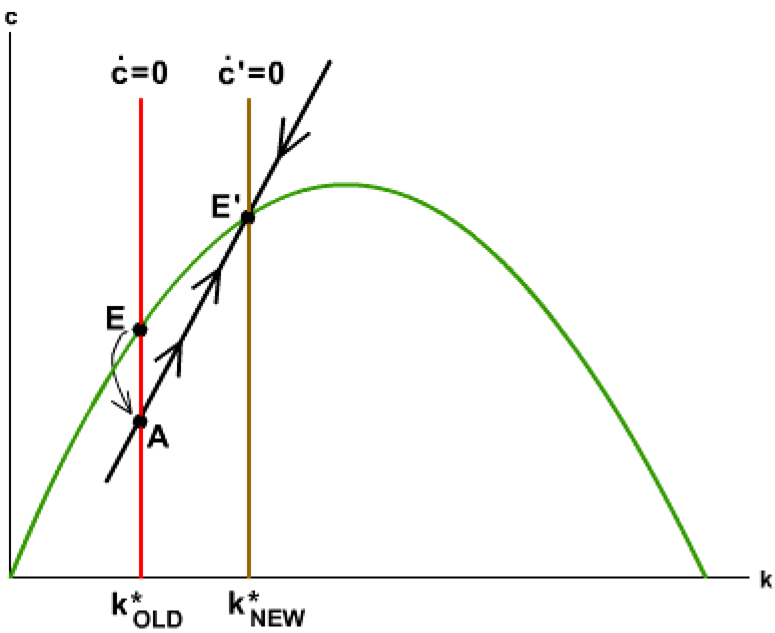




- The CKR model does not generate the long-run Golden Rule level of saving. People's discounting of the future leads them to save too little from a long-run welfare perspective. People's selfishness or shortsightedness reduces society's long-run welfare.

3.1.5 The effects of a fall in the discount rate

- Since the evolution of **k** is not determined by preference **θ**, only the $\dot{c} = 0$ locus is affected.



- At the instant of change, **c** jumps down so that the economy is on the new saddle path, say point A.

- Assume that a fall in θ is unexpected, the fall in $\mathbf{c}$ is the optimal response. The $\dot{c} = 0$ line shifts to the right.

- $\mathbf{k}$ rises gradually to a new higher level. $\mathbf{c}$ initially falls and rises to a level above the one it started at.

- The permanent fall in the discount rate produces temporary increases in the growth rate of capital per worker and output per worker.

3.1.6 The effects of permanent and temporary changes in government purchases

- Assume that the government buys output at the rate $\mathbf{G(t)}$ per unit time.

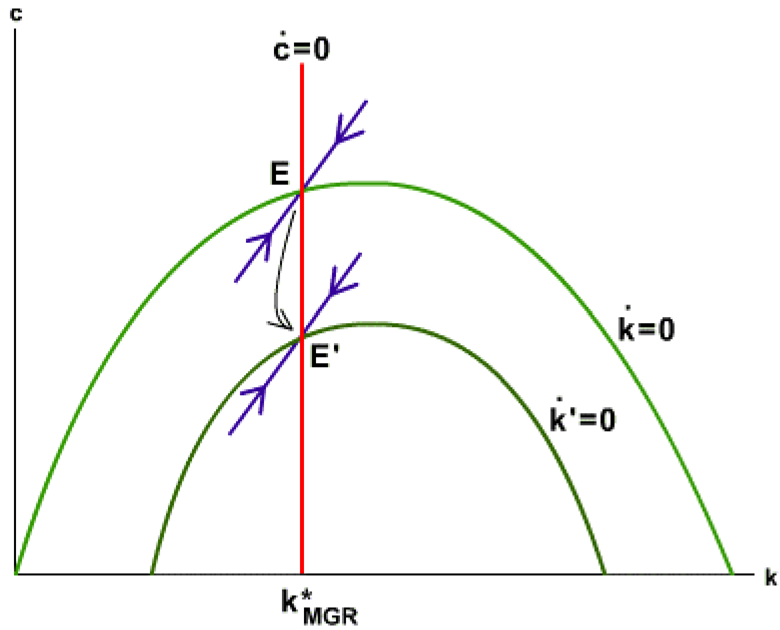
- Government purchases are assumed not to affect the marginal utility of consumption of the representative household.

- Government purchases are financed by lump-sum taxes of amount $\mathbf{G(t)}$ per unit time. Hence, the government always runs a balanced budget.

- Adding $\mathbf{G(t)}$ to **(1)**: $\dot{k} = f(k) - c - G - nk$ .... **(9)**

- A higher value of $\mathbf{G}$ shifts the $\dot{k} = 0$ locus down. The more goods that are purchased by the government, the fewer that can be purchased privately if $\mathbf{k}$ is to be held constant.

- Assume that there is an unexpected permanent increase in $\mathbf{G}$. The $\dot{k} = 0$ locus shifts down by the amount of the increase in $\mathbf{G}$.

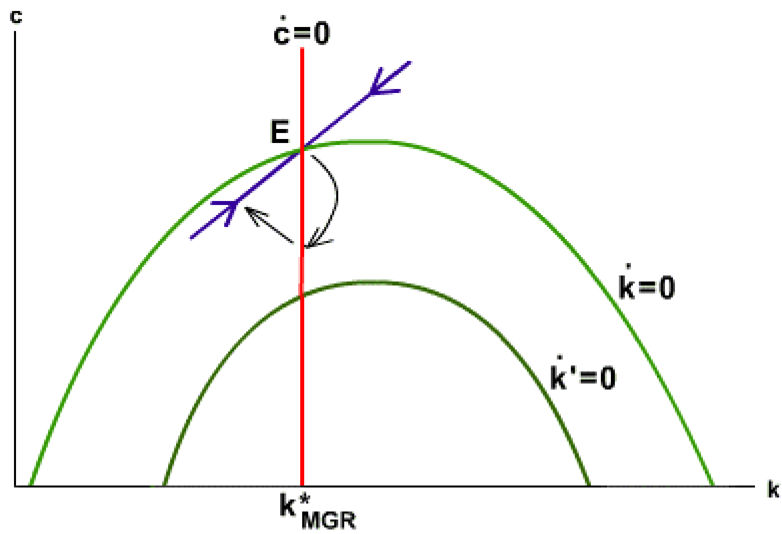


- The permanent increases in government purchases and taxes reduce households' lifetime wealth. Consumption falls immediately, and the capital stock and the real interest rate are unaffected.

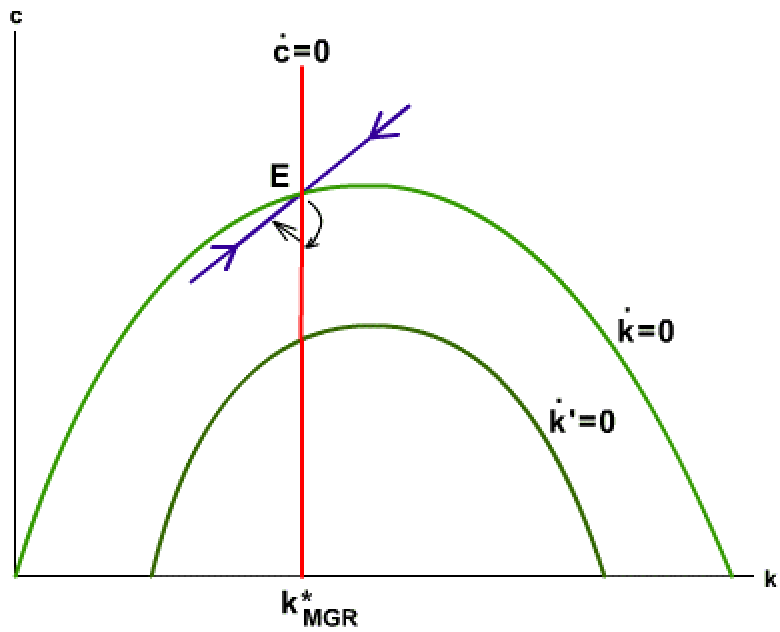
- Next, assume that increase in **G** is relatively long lasting. **c** falls by most of the amount of the increase in **G**.

- Assume that the economy is on a balanced growth path with **G(t)** constant at some level $\mathbf{G_L}$.

- As the time of the return of **G** to $\mathbf{G_L}$ approaches, households increase their consumption and decrease their capital holdings in anticipation of the fall in **G** .



- Assume that the increase in **G** is a short-lived rise. Households change their consumption relatively little. The effects on the capital stock and the real interest rate are small.



3.1.7 The CKR model and long-run growth

- The model assumes a positive rate of technological progress. Thus, economic growth is assumed.

- Economic growth will generate increases in rates of saving over time. But such increases are part of the transition.

- A higher rate of technological progress and long-run growth will lower the rate of saving in the steady state. The more the expectation of higher incomes in the future leads people to anticipate future income and consume today rather than waiting until the future.

- Like Solow model, technological progress can be introduced in the production function as labor - augmenting.

3.2 The empirical evidence on saving

- From the perspective of the population profile, the effect of faster economic growth on saving will tend to be positive because the people who gain the higher income are currently the people who do most of the saving.

- James Poterba shows that in most countries people save even after retirement. In Italy and Japan, the saving rate among elderly households, persons age 65 and older, actually exceeds 30%.

- Particularly in low-saving countries, household saving rates peak in the decade prior to retirement. Even in countries with social insurance and retirement benefits, the saving rate of individuals approaching retirement is high. Poterba explains this result as bequest motive.

- Statistical analysis confirms that overall saving is positively correlated with real per capita income, the rate of economic growth, and government saving (budget surpluses). But detailed analysis on direction of causality shows that economic growth seems to cause higher saving, but that higher saving does not cause permanently higher economic growth.

- The many empirical studies on saving still do not tell us much about how saving affects an economy's rate of growth. The evidence from Dani Rodrik also suggests that some countries get much more growth out of their saving than other countries do.

3.3 The financial sector and economic growth

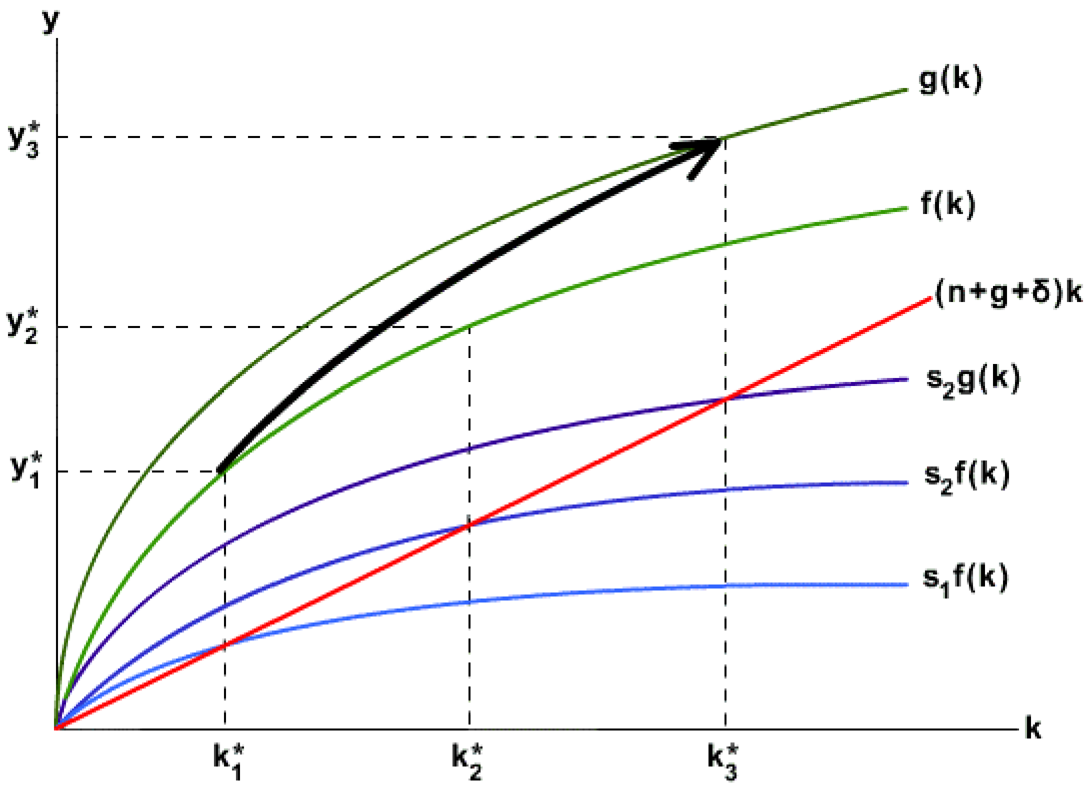
- John Hicks has suggested that financial markets should be considered as one of the causes of the surge in economic growth over the past 200 year.

- The decision to save is not equivalent to the decision to invest. Saving is the act of not consuming and storing wealth for future use, but investing involves using real resources to create new capital. The incentives that influence saving are not the same as those that motivate investment. It is the financial sector's job to channel the funds made available by savers to those who have the most productive investment projects.

3.3.1 The financial sector and the Solow model

- Improvements in the efficiency of the financial sector of the economy via the elimination of financial repression and a reduction in financial market failures can shift up the entire production function from **f(k)** to **g(k)**. This will increase saving further from $s_2f(k)$ to $s_2g(k)$. Hence, it is more possible to allocate savings to higher-return projects.

- The new steady-state levels of per-worker capital stock and per-worker output, $\mathbf{k_3^*}$ and $\mathbf{y_3^*}$, exceed not just the original levels, $\mathbf{k_1^*}$ and $\mathbf{y_1^*}$ but also the higher levels caused by the increase in saving and investment, $\mathbf{k_2^*}$ and $\mathbf{y_2^*}$.

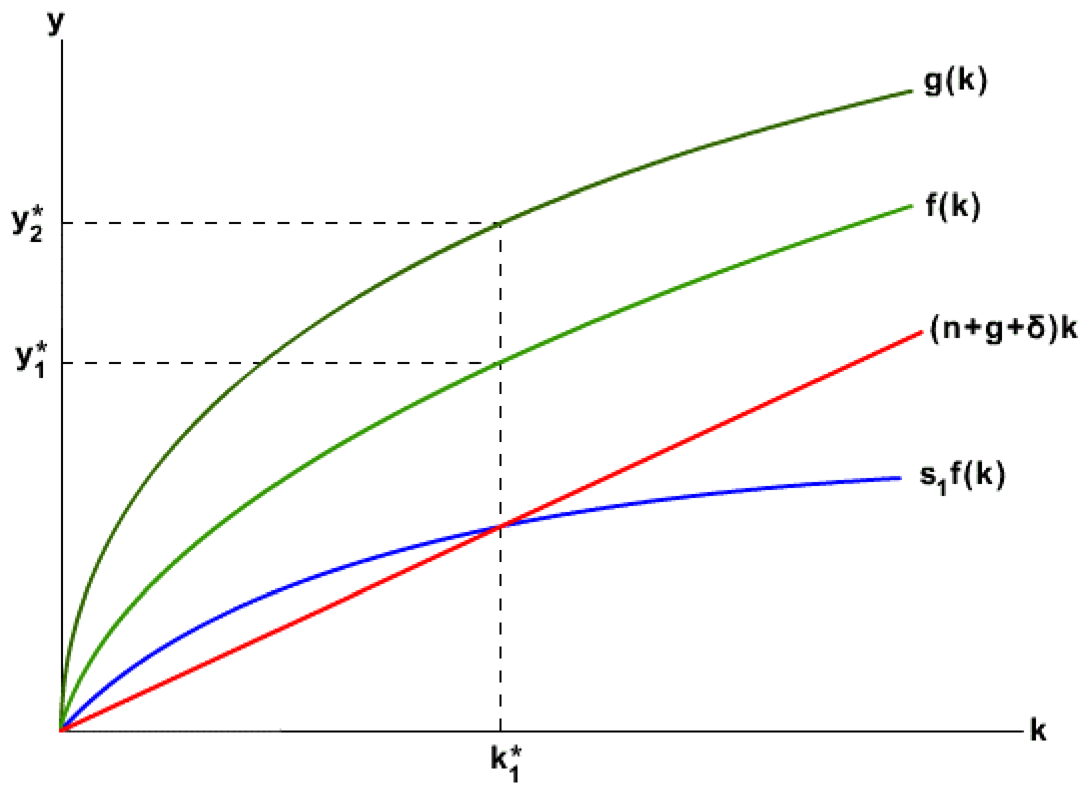


- The quality of investment can also improve because the financial sector becomes better at monitoring the performance of managers and firms. This also contributes to raising the production function.

- Empirical studies found that the increase in the level of saving and investment from financial liberalization might not be very large.

- If financial liberalization or an improvement in the performance of the financial sector does not increase saving, there is still a gain in per capita output from $\mathbf{y_1}$ to $\mathbf{y_2}$.

- The Solow model understates the gains from the improved performance of the financial sector. It captures only the short-run and medium-run effects of shifts in saving and improvements in the allocation of savings to investment projects. It does not explain the technological progress or long–run economic growth.



3.3.2 The financial sector and a standard R&D model

- Robert King and Ross Levine developed a model that adds financial intermediation to a standard model of research and development.

- The rate of technological progress **g** is a function of **L**, **π**, **β**, **r**, **f/γ** and **τ**. i.e. $\mathbf{g} = \mathbf{f}(\mathbf{L}, \pi, \beta, \mathbf{r}, \frac{\mathbf{f}}{\gamma}, \tau)$

- **L** stands for factors and resources contributed to innovation. A great number of people is create new products and to bring them into production and distribution. These people also need a lot of resources to help them innovate.

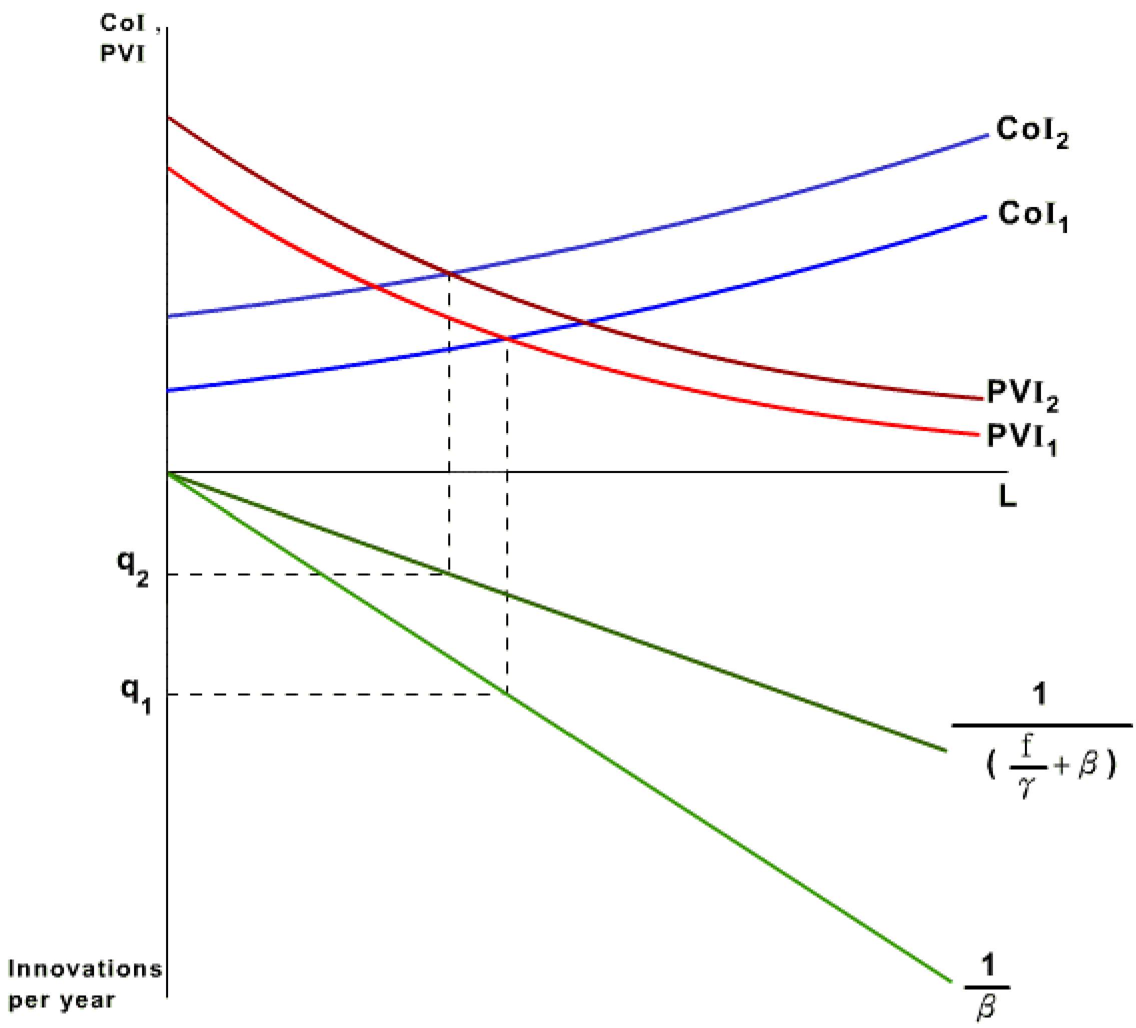
- **π** stands for a function of future profit, the length of time that an innovator enjoys the profit.

- **β** stands for the amount of resources needed to create an innovation. i.e. the amount of innovation, △Technology, is equal to **(1/β)L$_{R\&D}$**.

- **r** stands for the rate of interest with which future profit is discounted. Innovations require current expenditures to R&D that are paid for from future profits, thus the interest rate determines how future profits are discounted to the present. The higher the rate of interest **r**, the higher future profits must be to compensate innovators for the research and development costs.

- **f / γ** stands for the costs of investigating borrowers. **f** stands for the bank's resource cost of investigating each borrower and **γ** stands for the percentage of borrowers who are actually bonafrde entrepreneurs with profitable projects.

- **τ** stands for explicit taxes and costs imposed on financial intermediation by the government.



- The present value of innovation, **PVI**, is a function of future profit π, the rate of interest with which future profit is discounted **r**, and the amount of labor devoted to R&D.

$$PVI_{\pi} = f(\pi, r, \frac{L_{R\&D}}{\beta})$$

Where **PVIπ** = present value of future profit from innovation

- The **PVI** is a downward-sloping function of $L_{R\&D}$. It declines as the economy applies more resources to R&D activity because, ceteris paribus, profit will be destroyed more quickly. An increase in π or β raises the **PVI** curve, an increase in **r** lowers the **PVI** curve.

- The cost of innovation, **CoI**, is an upward-sloping function of $L_{R\&D}$. It increases as innovative activity increases. An increase in β shifts the **CoI** curve up.

- Given the **CoI** and **PVI** curves and the value of β, the economy will produce **q** innovations per year.

- A rise in f/γ reduces the number of innovations produces by a given amount of resources taken away from production, and thus the marginal cost of innovation increases from CoI_1 to CoI_2.

- The present value of innovation also increases from PVI_1 to PVI_2 because the higher costs of innovation imply that it will take longer for competitors to destroy an innovator's monopoly.

- Higher the costs of financial intermediation and the taxation of financial transactions will reduce the number of innovations production in the economy.